MATRICES

Precalculus
Chapter 9

This Slideshow was developed to accompany the textbook

- *Precalculus*
- *By Richard Wright*
- *https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html*

Some examples and diagrams are taken from the textbook.

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9-01 MATRICES AND SYSTEMS OF EQUATIONS

In this section, you will:

- Identify the order of a matrix.
- Write an augmented matrix for a system of equations.
- Write a matrix in row-echelon form.
- Solve a system of linear equations using an augmented matrix.

9-01 MATRICES AND SYSTEMS OF EQUATIONS

Matrix

- Rectangular array of numbers
- $\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$
- $a_{row, column}$
- Each entry is an element

- Augmented Matrix
  - Two matrices combined together

Order of matrix

- Dimension
- Rows × columns

What is the order of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$?

2×3

9-01 MATRICES AND SYSTEMS OF EQUATIONS

Elementary Row Operations

- Interchange 2 rows
- Multiply a row by a nonzero constant
- Add a multiple of a row to another row

Add 2 times 1st row to the 2nd row

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 9 & 12 \end{bmatrix}$$

9-01 MATRICES AND SYSTEMS OF EQUATIONS

Row-Echelon Form

- All rows consisting entirely of zeros are at bottom
- For other rows, the first nonzero entry is 1
- For successive rows, the leading 1 in the higher row is farther to the left

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reduced Row-Echelon Form

- Columns with leading 1 have 0’s as other entries

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9-01 MATRICES AND SYSTEMS OF EQUATIONS

Solve $\begin{cases} x + 3y + 4z = 7 \\ 2x + 7y + 5z = 10 \\ 3x + 10y + 4z = 27 \end{cases}$

$$\begin{bmatrix} 1 & 3 & 4 & \vdots & 7 \\ 2 & 7 & 5 & \vdots & 10 \\ 3 & 10 & 4 & \vdots & 27 \end{bmatrix}$$

-2×1st add to 2nd

$$\begin{bmatrix} 1 & 3 & 4 & \vdots & 7 \\ 0 & 1 & -3 & \vdots & -4 \\ 3 & 10 & 4 & \vdots & 27 \end{bmatrix}$$

-3×1st add to 3rd

$$\begin{bmatrix} 1 & 3 & 4 & \vdots & 7 \\ 0 & 1 & -3 & \vdots & -4 \\ 0 & 1 & -8 & \vdots & 6 \end{bmatrix}$$

-1×2nd add to 3rd

$$\begin{bmatrix} 1 & 3 & 4 & \vdots & 7 \\ 0 & 1 & -3 & \vdots & -4 \\ 0 & 0 & -5 & \vdots & 10 \end{bmatrix}$$

-1/5×3rd

$$\begin{bmatrix} 1 & 3 & 4 & \vdots & 7 \\ 0 & 1 & -3 & \vdots & -4 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

$$z = -2$$

$$y - 3z = -4 \rightarrow y = -10$$
$$x + 3y + 4z = 7 \rightarrow x = 45$$

(45, -10, -2)

9-02 GAUSSIAN ELIMINATION

In this section, you will:

- Write a matrix in reduced-row echelon form.
- Solve a system of linear equations using Gauss-Jordan Elimination.

9-02 GAUSSIAN ELIMINATION

Gaussian Elimination

- Solving a system of linear equations by putting it into row-echelon form with elementary row operations

Gauss-Jordan Elimination

- Solve by putting the system into Reduced row-echelon form

If a row becomes all zeros with final entry not zero = no solution

If a row becomes all zeros = many solutions (do the $z = a$ thing to write the parametric equations of the line of intersection)

9-02 GAUSSIAN ELIMINATION

Solve $\begin{cases} x \qquad - 3z = -5 \\ 3x + y - 2z = -4 \\ 2x + 2y + z = -2 \end{cases}$

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -5 \\ 3 & 1 & -2 & \vdots & -4 \\ 2 & 2 & 1 & \vdots & -2 \end{bmatrix}$$

-3×1^{st} add to 2^{nd}

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -5 \\ 0 & 1 & 7 & \vdots & 11 \\ 2 & 2 & 1 & \vdots & -2 \end{bmatrix}$$

-2×1^{st} add to 3^{rd}

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -5 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 2 & 7 & \vdots & 8 \end{bmatrix}$$

-2×2^{nd} add to 3^{rd}

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -5 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 0 & -7 & \vdots & -14 \end{bmatrix}$$

$-1/7\times 3^{rd}$

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -5 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

-7×3^{rd} add to 2^{nd}

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -5 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

3×3rd add to 1st

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

(1, -3, 2)

9-02 GAUSSIAN ELIMINATION

Solve $\begin{cases} x + y + 5z = -3 \\ -x - 2y - 8z = 5 \\ -x - 2z = 1 \end{cases}$

$$\begin{bmatrix} 1 & 1 & 5 & \vdots & -3 \\ -1 & -2 & -8 & \vdots & 5 \\ -1 & 0 & -2 & \vdots & 1 \end{bmatrix}$$

1st add to 2nd

$$\begin{bmatrix} 1 & 1 & 5 & \vdots & -3 \\ 0 & -1 & -3 & \vdots & 2 \\ -1 & 0 & -2 & \vdots & 1 \end{bmatrix}$$

1st add to 3rd

$$\begin{bmatrix} 1 & 1 & 5 & \vdots & -3 \\ 0 & -1 & -3 & \vdots & 2 \\ 0 & 1 & 3 & \vdots & -2 \end{bmatrix}$$

2nd add to 3rd

$$\begin{bmatrix} 1 & 1 & 5 & \vdots & -3 \\ 0 & -1 & -3 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

Many solutions

-1×2nd

$$\begin{bmatrix} 1 & 1 & 5 & \vdots & -3 \\ 0 & 1 & 3 & \vdots & -2 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

-1×2^{nd} add to 1^{st}

$$\begin{bmatrix} 1 & 0 & 2 & \vdots & -1 \\ 0 & 1 & 3 & \vdots & -2 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$z = a$$

$$y + 3z = -2 \rightarrow y = -2 - 3a$$

$$x + 2z = -1 \rightarrow x = -1 - 2a$$

(-1-2a, -2-3a, a)

9-03 MATRIX OPERATIONS

In this section, you will:

- Add and subtract matrices.
- Multiply a scalar with a matrix.
- Multiply a matrix with a matrix.

9-03 MATRIX OPERATIONS

Matrix addition and subtraction

- Both matrices must have same order
- Add or subtract corresponding elements

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -2 & -3 \\ -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 3+0 & 1+(-1) \\ 0+(-2) & 2+(-3) \\ -4+(-4) & -1+(-5) \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ -2 & -1 \\ -8 & -6 \end{bmatrix}$$

9-03 MATRIX OPERATIONS

Scalar multiplication

- Multiply a matrix with a number
- Distribute

$$3\begin{bmatrix}1 & 2 & 3\\0 & -1 & -2\end{bmatrix}$$

$$\begin{bmatrix}3\cdot 1 & 3\cdot 2 & 3\cdot 3\\3\cdot 0 & 3\cdot -1 & 3\cdot -2\end{bmatrix}$$
$$\begin{bmatrix}3 & 6 & 9\\0 & -3 & -6\end{bmatrix}$$

9-03 MATRIX OPERATIONS

Matrix multiplication

- Number of columns in 1st = number of rows in 2nd
- $(m \times n) \cdot (n \times p)$
- Order of product $m \times p$
- Order is important
- NO COMMUTATIVE PROPERTY!!!!!

9-03 MATRIX OPERATIONS

$$\begin{bmatrix} 2 & -1 & 7 \\ 0 & 6 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 \cdot 0 + (-1)(-2) + 7 \cdot 3 \\ 0 \cdot 0 + 6 \cdot (-2) + (-3) \cdot 3 \end{bmatrix}$$

$$\begin{bmatrix} 23 \\ -21 \end{bmatrix}$$

9-03 MATRIX OPERATIONS

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 4 \\ -2 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2(-1) + 0(-2) & 2(0) + 0(1) & 2(4) + 0(2) \\ 1(-1) + 3(-2) & 1(0) + 3(1) & 1(4) + 3(2) \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 8 \\ -7 & 3 & 10 \end{bmatrix}$$

9-04 INVERSE MATRICES

In this section, you will:

- Find the inverse of a square matrix
- Use the inverse of a matrix to solve a matrix equation.

9-04 INVERSE MATRICES

Identity Matrix (I)

- $A \cdot I = A$
- $A \cdot A^{-1} = I$
- Both A and A^{-1} must be square

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

OR

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

OR

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9-04 INVERSE MATRICES

Inverse of 2×2

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find the inverse of $\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$

$$\frac{1}{1(4) - (-2)(0)}\begin{bmatrix} 4 & -0 \\ 2 & 1 \end{bmatrix}$$

$$\frac{1}{4}\begin{bmatrix} 4 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1/2 & 1/4 \end{bmatrix}$$

9-04 INVERSE MATRICES

Find other inverses

- Augment the matrix with the identity matrix
- Use Gauss-Jordan elimination to turn the original matrix into the identity matrix

$$[A \vdots I] \rightarrow [I \vdots A^{-1}]$$

9-04 INVERSE MATRICES

Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ -3 & 4 & -4 \end{bmatrix}$

Augment with identity

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 1 & 0 \\ -3 & 4 & -4 & 0 & 0 & 1 \end{array}\right]$$

3×1st add to 3rd

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 1 & 0 \\ 0 & 10 & 5 & 3 & 0 & 1 \end{array}\right]$$

10×2nd add to 3rd

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 1 & 0 \\ 0 & 0 & -15 & 3 & 10 & 1 \end{array}\right]$$

-2×3rd add to 15×2nd

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -15 & 0 & -6 & -5 & -2 \\ 0 & 0 & -15 & 3 & 10 & 1 \end{array}\right]$$

3rd add to 5×1st

$$\left[\begin{array}{ccc|ccc} 5 & 10 & 0 & 8 & 10 & 1 \\ 0 & -15 & 0 & -6 & -5 & -2 \\ 0 & 0 & -15 & 3 & 10 & 1 \end{array}\right]$$

2×2nd add to 3×1st

$$\begin{bmatrix} 15 & 0 & 0 & \vdots & 12 & 20 & -1 \\ 0 & -15 & 0 & \vdots & -6 & -5 & -2 \\ 0 & 0 & -15 & \vdots & 3 & 10 & 1 \end{bmatrix}$$

1/15×1st, -1/15×2nd, -1/15×3rd

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 4/5 & 4/3 & -1/15 \\ 0 & 1 & 0 & \vdots & 2/5 & 1/3 & 2/15 \\ 0 & 0 & 1 & \vdots & -1/5 & -2/3 & -1/15 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{5} & \frac{4}{3} & -\frac{1}{15} \\ \frac{2}{5} & \frac{1}{3} & \frac{2}{15} \\ -\frac{1}{5} & -\frac{2}{3} & -\frac{1}{15} \end{bmatrix}$$

9-04 INVERSE MATRICES

Use an inverse to solve system of equations

Write system as matrices

$AX = B$ (coefficients · variables = constants)

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

Solve by multiplying the inverse of the coefficients with the constants

9-04 INVERSE MATRICES

Solve $\begin{cases} 2x + 3y = 0 \\ x - 4y = 7 \end{cases}$

$$\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

Find inverse of coefficients

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 1 & -4 & 0 & 1 \end{array}\right]$$

1st add to -2×3rd

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 0 & 11 & 1 & -2 \end{array}\right]$$

-3×2nd add to 11×1st

$$\left[\begin{array}{cc|cc} 22 & 0 & 8 & 6 \\ 0 & 11 & 1 & -2 \end{array}\right]$$

1/22×1st, 1/11×2nd

$$\left[\begin{array}{cc|cc} 1 & 0 & 4/11 & 3/11 \\ 0 & 1 & 1/11 & -2/11 \end{array}\right]$$

Multiply inverse with constants

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & -\frac{2}{11} \end{bmatrix} \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{21}{11} \\ -\frac{14}{11} \end{bmatrix}$$

(21/11, -14/11)

9-05 DETERMINANTS OF MATRICES

In this section, you will:

- Find a determinant 2×2 or 3×3 matrix using shortcuts.
- Find a determinant of any square matrix using expansion by cofactors.

9-05 DETERMINANTS OF MATRICES

Determinant is a real number associated with a square matrix

Find $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

2×2

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then
- $\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$
- $= ad - bc$

Down product – up product

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$
$$= 1(4) - 3(2)$$
$$= 4 - 6$$
$$= -2$$

9-05 DETERMINANTS OF MATRICES

3×3

- Copy 1st two columns after matrix
- + products of downs − products of ups

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}\begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$$

$$= 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8 - 7 \cdot 5 \cdot 3 - 8 \cdot 6 \cdot 1 - 9 \cdot 4 \cdot 2$$

$$= 45 + 84 + 96 - 105 - 48 - 72$$

$$= 0$$

9-05 DETERMINANTS OF MATRICES

Otherwise

- Expansion by cofactors

Sign Pattern

- $\begin{bmatrix} + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$

Minor

- Determinant of matrix created by crossing out a row and column

Cofactor

- Minor with sign from sign pattern

Given $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$, find

Minor M_{13}

Cofactor C_{13}

Minor $M_{13} = \begin{bmatrix} \not{1} & \not{0} & \not{3} \\ 2 & 1 & \not{0} \\ 0 & 2 & \not{3} \end{bmatrix}$

$$\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix}$$
$$= 4 - 0 = 4$$

Cofactor $C_{13} = +(4) = 4$

9-05 DETERMINANTS OF MATRICES

Find $\begin{vmatrix} -1 & 0 & 4 \\ 3 & -2 & 0 \\ 1 & -1 & 1 \end{vmatrix}$

Pick a row or column with 0's like 1st row
Find all the cofactors of 1st row

$$\begin{vmatrix} -1 & 0 & 4 \\ 3 & -2 & 0 \\ 1 & -1 & 1 \end{vmatrix}$$
$$= +(-1) \cdot \begin{vmatrix} -2 & 0 \\ -1 & 1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix} + 4 \begin{vmatrix} 3 & -2 \\ 1 & -1 \end{vmatrix}$$
$$= -1(-2 - 0) - 0(\quad) + 4\big(-3 - (-2)\big)$$
$$= 2 + (-4)$$
$$= -2$$

9-05 DETERMINANTS OF MATRICES

Find $\begin{vmatrix} -2 & 4 & 0 & 5 \\ 0 & 2 & -1 & 0 \\ 3 & 1 & -4 & -1 \\ -5 & 0 & -2 & 3 \end{vmatrix}$

Pick 2[nd] row

$$-0|\ | + 2\begin{vmatrix} -2 & 0 & 5 \\ 3 & -4 & -1 \\ -5 & -2 & 3 \end{vmatrix} - (-1)\begin{vmatrix} -2 & 4 & 5 \\ 3 & 1 & -1 \\ -5 & 0 & 3 \end{vmatrix} + 0|\ |$$

$$= -0(\quad) + 2(24 + 0 + (-30) - 100 - (-4) - 0)$$
$$+ 1(-6 + 20 + 0 - (-25) - 0 - 36) + 0(\quad)$$
$$= 2(-102) + 1(3)$$
$$= -201$$

9-06 APPLICATIONS OF MATRICES

In this section, you will:

- Solve a System of Linear Equations by Cramer's Rule.
- Use a determinant to find the area of a triangle.
- Use a determinant to determine if three points are collinear.
- Use a determinant to find the equation of a line.
- Use a matrix to encode and decode a message.

9-06 APPLICATIONS OF MATRICES

Cramer's Rule

- Used to solve systems of equations
- $x_1 = \frac{|A_1|}{|A|}$ $\qquad$ $x_2 = \frac{|A_2|}{|A|}$
  - A = coefficient matrix
  - A_n = coefficient matrix with column n replaced with constants
- If |A| = 0, then no solution or many solutions

9-06 APPLICATIONS OF MATRICES

Use Cramer's Rule

$$\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$$

$$x = \frac{\begin{vmatrix} 6 & 1 & 1 \\ 1 & -1 & 3 \\ -3 & 1 & -2 \end{vmatrix}\begin{matrix} 6 & 1 \\ 1 & -1 \\ -3 & 1 \end{matrix}}{\begin{vmatrix} 2 & 1 & 1 \\ -1 & -1 & 3 \\ 0 & 1 & -2 \end{vmatrix}\begin{matrix} 2 & 1 \\ -1 & -1 \\ 0 & 1 \end{matrix}} = \frac{12 + (-9) + 1 - 3 - 18 - (-2)}{4 + 0 + (-1) - 0 - 6 - 2} = -\frac{15}{-5} = 3$$

$$y = \frac{\begin{vmatrix} 2 & 6 & 1 \\ -1 & 1 & 3 \\ 0 & -3 & -2 \end{vmatrix}\begin{matrix} 2 & 6 \\ -1 & 1 \\ 0 & -3 \end{matrix}}{-5} = \frac{-4 + 0 + 3 - 0 - (-18) - 12}{-5} = \frac{5}{-5} = -1$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 6 \\ -1 & -1 & 1 \\ 0 & 1 & -3 \end{vmatrix}\begin{matrix} 2 & 1 \\ -1 & -1 \\ 0 & 1 \end{matrix}}{-5} = \frac{6 + 0 + (-6) - 0 - 2 - 3}{-5} = \frac{-5}{-5} = 1$$

(3, -1, 1)

9-06 APPLICATIONS OF MATRICES

Area of triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3)

$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Find the area of triangle with vertices (-3, 1), (2, 4), (5, -3)

$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$Area = \pm\frac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ 2 & 4 & 1 \\ 5 & -3 & 1 \end{vmatrix}\begin{matrix} -3 & 1 \\ 2 & 4 \\ 5 & -3 \end{matrix}$$

$$= \pm\frac{1}{2}((-12 + 5 + (-6) - 20 - 9 - 2)$$

$$= \pm\frac{1}{2}(-44)$$

$$= 22$$

9-06 APPLICATIONS OF MATRICES

Lines in a Plane

- If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$, then the points are collinear

- Find equation of line given 2 points (x_1, y_1) and (x_2, y_2)
  - $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Find the equation of the line passing through (-2, 9) and (3, -1)

$$0 = \begin{vmatrix} x & y & 1 \\ -2 & 9 & 1 \\ 3 & -1 & 1 \end{vmatrix} \begin{matrix} x & y \\ -2 & 9 \\ 3 & -1 \end{matrix}$$

$$9x + 3y + 2 - 27 - (-x) - (-2y) = 0$$

$$10x + 5y - 25 = 0$$

$$2x + y - 5 = 0$$

9-06 APPLICATIONS OF MATRICES

Hill Cypher **Encoding a Message**

1. Convert the message into numbers
2. Choose a square encoding matrix.
3. Group the message numbers into matrices of 1 row and the same number of columns as the encoding matrix.
4. Multiply the letter matrices with the encoding matrix.
5. The encoded message is the list of numbers produced.

- **Decode** by using inverse of encoding matrix

_ = 0	I = 9	R = 18
A = 1	J = 10	S = 19
B = 2	K = 11	T = 20
C = 3	L = 12	U = 21
D = 4	M = 13	V = 22
E = 5	N = 14	W = 23
F = 6	O = 15	X = 24
G = 7	P = 16	Y = 25
H = 8	Q = 17	Z = 26

9-06 APPLICATIONS OF MATRICES

Encode LUNCH using $\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$

Letters become
12, 21, 14, 3, 8, 0

$$[12 \quad 21]\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix} = [54 \quad -63]$$
$$[14 \quad 3]\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix} = [20 \quad -9]$$
$$[8 \quad 0]\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix} = [8 \quad 0]$$

Message: 24, -63, 20, -9, 8, 0